

MHF4U

Reach Ahead

PRACTICE Examination

July 23rd, 2015

Length: 2 hrs + 1 hr flex time



OTTAWA-CARLETON
DISTRICT SCHOOL BOARD
SUMMER SCHOOL

Name : _____

Teacher : MS K. VLUG

School : ADULT HIGH SCHOOL

Instructions to students:

1. This examination booklet is **12 pages**.
Please check that you have all the pages.
2. Answer all questions with complete solutions in the spaces provided on the examination paper. **PLEASE CIRCLE YOUR FINAL ANSWERS.**
3. You may use any school-approved calculator on this examination.
Do **not** share your calculator.
4. There is a formula page attached at the end of this exam.

Unit	Level Achieved
Unit A – Exponentials and Logs	
Unit B – Trigonometry	
Unit C – Polynomial and Rational Functions	
Unit D – Characteristics of Functions	

Evaluation Rubric

Criteria	R Insufficient	Level 1 Limited	Level 2 Approaching	Level 3 Sufficient	Level 4 Thorough Sophisticated
Knowledge and Understanding					
Understanding of content and concepts	Insufficient understanding of content and concepts.	Simplistic understanding of content and concepts. Many conceptual gaps or errors.	Basic understanding of content and concepts. Some conceptual gaps or errors	Sufficient understanding of content and concepts. Rare conceptual gaps or errors.	Insightful understanding of content and concepts. No significant conceptual errors
Procedural accuracy		Mathematics contains many procedural errors	Execution of mathematics contains some procedural errors.	Mathematics may contain a few procedural errors	Mathematics may contain insignificant or rare procedural errors
Communication					
Expression and organization of ideas and mathematical thinking	Solution unorganized . Insufficient use of correct mathematical vocabulary, symbols, labels, and conventions.	Organization of mathematics poor . Uses common language in place of mathematical vocabulary. Sometimes uses symbols, labels, and conventions correctly.	Organization of mathematics satisfactory . Few errors in vocabulary and only some common language in place of mathematical vocabulary. Usually uses mathematical symbols, labels, and conventions correctly.	Organization of mathematics easy to follow . Appropriate use of mathematical vocabulary . Consistently uses mathematical symbol, labels, and conventions correctly.	Organization of mathematics clear and easy to follow . Clear and precise language. Insightful use of symbols and mathematical vocabulary.
Application					
Transfer of mathematical ideas to situations drawn from other contexts	Insufficient transfer of ideas to other contexts.	Simplistic transfer of ideas drawn from familiar contexts.	Basic transfer of ideas drawn from familiar contexts .	Solid transfer of ideas drawn from new contexts .	Transfers ideas to other contexts and makes unique, original, or insightful connections.
Selection and use of tools and strategies to solve a problem	Incorrect selection of tools	Selection of appropriate tools is limited , with major errors, omissions or mis-sequencing.	Selects some appropriate tools and strategies with minor errors, omissions or mis-sequencing.	Selects appropriate tools and strategies accurately and in logical sequence.	Selects the most appropriate tools accurately and efficiently. Creates elegant solutions , uses multiple strategies.
Connections between representations	Insufficient connections made between different representations.	Makes few simple connections . Limited use of multiple representations.	Makes few basic connections . Basic use of multiple representations Misinterprets part(s) of the information	Makes solid connections . Appropriate use of multiple representations .	Makes strong connections between representations . Insightful use of multiple representations as appropriate to the problem.
Reasoning with justification.	Insufficient evidence of reasoning.	Simplistic reasoning with weak justification and possible generalizations .	Basic evidence of reasoning with minor errors in justification and possible generalizations	Reasoning of familiar situations with justification and some generalizations .	Sophisticated mathematical reasoning with strong justification and generalizations

Unit A – Exponentials and Logs

1.) Given that $y = -5\log_2(x - 3) + 9$, state:

- a) The domain _____
- b) Equation of the asymptote _____
- c) X, if $P(X, -1)$ _____

3.) A small town committee estimates that the population (P) of the town will change with respect to time, t, in years, according to the equation $P = 3000\sqrt{3+t}$. Estimate the rate at which the population will be increasing with respect to time at 3 years. Show all your calculations.

4.) Evaluate each of the following:

- (a) $\log_3 27$ _____
- (b) $\log_2 \frac{1}{16}$ _____
- (c) $\log_5 \sqrt[3]{25}$ _____
- (d) $\log_{2.5} 1$ _____
- (e) $\log_7 0$ _____
- (f) $\log_7 4$ (to 4 decimal places) _____

5. Solve x, $x \in \mathbb{R}$:

- (a) $\log_x 16 = \frac{4}{5}$
- (b) $\log_9 3\sqrt{3} = x$
- (c) $45 = 3.5(2.4)^{3x}$. **{4d.p}**
- (d) $5^{x+3} = 8^{x-3}$. **{4d.p}**
- (e) $\log_3(x - 5) + \log_3(x - 3) = 1$

6.) The amount of a certain medication decreases by 16% per hour in the bloodstream. A patient was injected with 100 ml of the medication at 8:00 a.m.

- (a) Write an equation to determine the amount of the medication in the bloodstream t hours after it was administered. Include proper “Let ...” statements to introduce your variable.
- (b) At 4:00 p.m. of the same day the patient will be administered the second dosage of the medication. How much of the first dosage is left in the bloodstream at 4:00 p.m.?
- (c) At what time, between 8:00 a.m. and 4:00 p.m. would the amount of the medication from the first dosage be half of what was administered ? Correct your answer to the nearest minute.

Unit B – Trigonometry

7.) Graph each trigonometric function for one period.

a) $y = -5 \csc(3x) + 1$

b) $y = -3 \cot(x + \pi) + 2$

8.) If $\cos x = \frac{4}{5}$ and $\cos \alpha = \frac{5}{13}$ where x and α are both acute, find $\sin(\alpha - x)$.

11.) Prove the following identities

a) $\frac{1}{1 - \sec \theta} + \frac{1}{1 + \sec \theta} = -2 \cot^2 \theta$

b) $\frac{\csc^2 \theta - 2}{\csc^2 \theta} = \cos 2\theta$

c) $\frac{\cos(x + y)}{\sin x \cos y} = \cot x - \tan y$

d) $\frac{1 - \sin 2x}{\cos 2x} = \frac{\cos 2x}{1 + \sin 2x}$

e) $\tan(x + y + z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan x \tan z}$

12.) Find the exact value of the following.

a) $\tan\left(\frac{3\pi}{4}\right)$

b) $\csc\left(\frac{-5\pi}{6}\right)$

c) $\cot\left(\frac{5\pi}{3}\right)$

Unit C – Polynomial and Rational Functions

13.) A manufacturer has a 0.5 oz tolerance for a bottle of salad dressing advertised as 189 oz. Write and solve an absolute value inequality that describes the acceptable volumes for “189 oz” bottles.

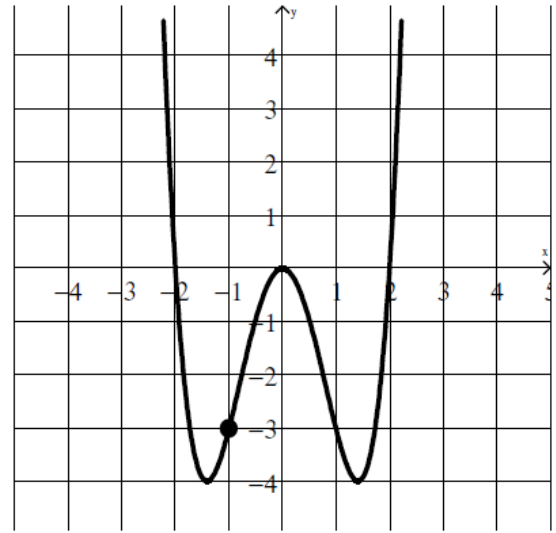
14.) Solve $\frac{x-5}{x+1} = \frac{x^2 + 2x - 4}{x+3}$

15.) When $x^4 - 4x^3 + ax^2 + bx + 1$ is divided by $(x-3)$, the remainder is -9. When it is divided by $(x+2)$, the remainder is 5. Determine the values of a and b .

16.) Explain the difference between $(4,8)$ and $(4,8]$

17.) graph $g(x) = \frac{x^2 + 3x + 2}{x^2 + x - 2}$

18.) Determine an equation of the function below:



19.) Use long division to determine the number of real roots and factor the function as far as possible, given $f(x) = x^3 - 8x^2 + 5x - 3$. State the roots.

20.) Solve $x^3 - 5x^2 + 7x - 3 \leq 0$

21.) Sketch each rational function

a) $\frac{x^2 - 1}{x + 2}$ b) $\frac{3x - 1}{x + 2}$ c) $\frac{-1}{x^2 + 2}$

Unit D – Characteristics of Functions

22.) Let $f = \{(1,2), (3,4), (5,6), (7,8)\}$ and $g = \{(1,5), (3,7), (4,0), (7,7)\}$, determine

a) $f + g$

b) $f - g$

c) fg

23.) Complete the following table

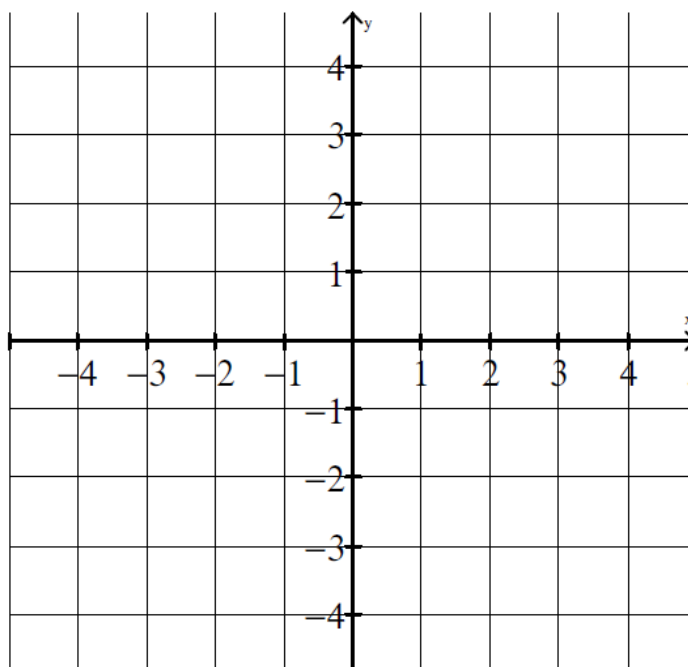
	$y = 2^x$	$y = 2x^3$	$y = \frac{1}{x+6}$	$y = \sqrt{(x+3)}$
Sketch				
Domain				
Range				
End Behaviours				
Equations of Asymptotes				
Symmetry				

24.) Determine if the piecewise function given below is continuous.

$$f(x) = \begin{cases} 3, & \text{if } x \leq -2 \\ x+5, & \text{if } -2 < x \leq 3 \\ x^2 - 2, & \text{if } x > 3 \end{cases}$$

25.) Graph the following relation

$$|x - y| \geq 2$$



26.) Given: $f(x) = x + 5$ and $g(x) = x^2 + 8$

Find:

a) $(g - f)(3)$

b) $\left(\frac{g}{f}\right)(x)$

c) $f^{-1}(x)$

d) $(fg)(-1)$

e) $(g \circ f)(5)$

27.) Given $f(x) = 2^x$ and $g(x) = 3x - 4$, find $g \circ f(x)$ and state the domain and range of $g \circ f(x)$